

Re-examination of the Perturbative Pion Form Factor with Sudakov Suppression

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The perturbative pion form factor with Sudakov suppression is re-examined. Taking into account the multi-gluon exchange in the low Q^2 regions, we suggest that the running coupling constant should be frozen at $\alpha_s(t = \sqrt{\langle \mathbf{k}_T^2 \rangle})$ and $\sqrt{\langle \mathbf{k}_T^2 \rangle}$ is the average transverse momentum which can be determined by the pionic wave function. In addition, we correct the previous calculations about the Sudakov suppression factor which plays an important role in the perturbative predictions for the pion form factor.

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It is believed that perturbative QCD (pQCD) can successfully describe exclusive processes at asymptotically large momentum transfers [1]. However, the applicability of the pQCD to the pion form factor at the present energy is a matter of controversy [2]. Recent studies [3-5] on the pion electromagnetic form factor show that the pQCD contributions become self-consistent for momentum transfers at a few GeV range. Li and Sterman [4] give a modified expression for the pion form factor by taking into account the customarily neglected partonic transverse momenta as well as the Sudakov corrections. Jakob and Kroll [5] point out that the dependence of the hadronic wave function on the intrinsic transverse momentum should be considered in the perturbative calculation. Sudakov corrections come from an infinite summation of higher-order effects associated with the elastic scattering of the valence partons. However, because the running coupling constant α_s becomes rather large with b (the distance between a quark-antiquark pair) increasing in the end-point regions, a cut-off on α_s has to be made to evaluate perturbative contributions and to justify the self-consistency of perturbative calculations. In this paper, we re-examine the perturbative pion form factor with the Sudakov suppression. It is pointed out that $\alpha_s(t)$ should be frozen as t is smaller than a certain value due to the multi-gluon exchange at low Q^2 . We suggest that the frozen point is related to the average transverse momentum which is determined by the pionic wave function. In addition, we correct the previous calculations about the Sudakov suppression factor which plays an important role in the perturbative predictions for the pion form factor.

Let us begin with a brief review on the derivation of the expression for the pion form factor in Ref. [4] Taking into account the transverse momenta \mathbf{k}_T that flow from the wave functions through the hard scattering leads to a factorization form with two wave functions $\psi_i(x_i, \mathbf{k}_{T_i})$ corresponding to the external pions, combined with a new hard-scattering function $T_H(x_1, x_2, Q, \mathbf{k}_{T_1}, \mathbf{k}_{T_2})$, which depends in general on transverse as well as longitudinal momenta,

$$F_\pi(Q^2) = \int_0^1 dx_1 dx_2 \int d^2\mathbf{k}_{T_1} d^2\mathbf{k}_{T_2} \psi_i(x_1, \mathbf{k}_{T_1}, P_1)$$

$$\times T_H(x_1, x_2, Q, \mathbf{k}_{T_i}, \mu) \psi_i(x_2, \mathbf{k}_{T_2}, P_2), \quad (1)$$

where $Q^2 = 2P_1 \cdot P_2$, and μ is the renormalization and factorization scale.

Through Fourier transformation eq. (1) can be expressed as

$$F_\pi(Q^2) = \int_0^1 dx_1 dx_2 \frac{d\mathbf{b}_1}{(2\pi)^2} \frac{d\mathbf{b}_2}{(2\pi)^2} \varphi(x_1, \mathbf{b}_1, P_1, \mu) \\ \times T_H(x_1, x_2, Q, \mathbf{b}_1, \mathbf{b}_2, \mu) \varphi(x_2, \mathbf{b}_2, P_2, \mu). \quad (2)$$

In this expression, wave function $\varphi(x_i, \mathbf{b}_i, P_i, \mu)$ takes into account an infinite summation of higher-order effects associated with the elastic scattering of the valence partons, which gives out the Sudakov suppression to the large- b and small- x regions. At the same time the intrinsic transverse momentum dependence of the wave function provides another suppression to the large- b regions.

At the lowest order, T_H is given by

$$T_H(x_1, x_2, Q, \mathbf{k}_{T_i}) = \frac{16\pi C_F \alpha_s(\mu)}{x_1 x_2 Q^2 + (\mathbf{k}_{T_1} + \mathbf{k}_{T_2})^2}, \quad (3)$$

where C_F is the color factor. The wave function can be expressed as

$$\varphi(x, b, P, \mu) = \exp \left[-s(x, b, Q) - s(1-x, b, Q) - 2 \int_{1/b}^\mu \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(g(\bar{\mu})) \right] \times \phi \left(x, \frac{1}{b} \right), \quad (4)$$

where $\gamma_q = -\alpha_s/\pi$ is the quark anomalous dimension in the axial gauge. $s(\xi, b, Q)$ is the Sudakov exponent factor, which reads

$$s(\xi, b, Q) = \frac{A^{(1)}}{2\beta_1} \hat{q} \ln \left(\frac{\hat{q}}{-\hat{b}} \right) + \frac{A^{(2)}}{4\beta_1^2} \left(\frac{\hat{q}}{-\hat{b}} - 1 \right) - \frac{A^{(1)}}{2\beta_1} (\hat{q} + \hat{b}) \\ - \frac{A^{(1)}\beta_2}{4\beta_1^3} \hat{q} \left[\frac{\ln(-2\hat{b}) + 1}{-\hat{b}} - \frac{\ln(-2\hat{q}) + 1}{-\hat{q}} \right] \\ - \left(\frac{A^{(2)}}{4\beta_1^2} - \frac{A^{(1)}}{4\beta_1} \ln \left(\frac{1}{2} e^{2\gamma-1} \right) \right) \ln \left(\frac{\hat{q}}{-\hat{b}} \right) \\ + \frac{A^{(1)}\beta_2}{8\beta_1^3} \left[\ln^2(2\hat{q}) - \ln^2(-2\hat{b}) \right], \quad (5)$$

where

$$\hat{q} = \ln[\xi Q/(\sqrt{2}\Lambda)], \quad \hat{b} = \ln(b\lambda),$$

$$\beta_1 = \frac{33 - 2n_f}{12}, \quad \beta_2 = \frac{153 - 19n_f}{24},$$

$$A^{(1)} = \frac{4}{3}, \quad A^{(2)} = \frac{67}{9} - \frac{1}{3}\pi^2 - \frac{10}{27}n_f + \frac{8}{3}\beta_1 \ln\left(\frac{1}{2}e^\gamma\right), \quad (6)$$

where n_f is the number of quark flavors and γ is the Euler constant.

It should be noted that there are some mistakes in the coefficients of the fourth and the sixth terms in $s(\xi, b, Q)$ given by Refs. [4,6]. We find that the correct coefficients should be $-\frac{A^{(1)}\beta_2}{4\beta_1^3}$ and $+\frac{A^{(1)}\beta_2}{8\beta_1^3}$, in place of $-\frac{A^{(1)}\beta_2}{16\beta_1^3}$ and $-\frac{A^{(1)}\beta_2}{32\beta_1^3}$ in Refs. [4,6]. It is $s(\xi, b, Q)$ that play an important role in the evaluation of the pion form factor. In this paper, we examine the effects brought about by these corrections.

Applying the renormalization group equation to T_H and substituting the explicit expression for T_H , we have the following expression for the pion form factor

$$F_\pi(Q^2) = 16\pi C_F \int_0^1 dx_1 dx_2 \int_0^\infty b db \alpha_s(t) K_0(\sqrt{x_1 x_2} Q b)$$

$$\times \phi(x_1, 1/b) \phi(x_2, 1/b) \exp(-S(x_1, x_2, Q, b, t)), \quad (7)$$

where

$$S(x_1, x_2, Q, b, t) = \left[\sum_{i=1}^2 (s(x_i, b, Q) + s(1 - x_i, b, Q)) - \frac{2}{\beta_1} \ln \frac{\hat{t}}{-\hat{b}} \right]. \quad (8)$$

Radiative corrections in higher orders will bring logarithms of the form $\ln(t/\mu)$ into T_H , where t is the largest mass scale appearing in T_H . Ref. [4] points out that a natural choice for μ in T_H is $\mu = t$ and

$$t = \max(\sqrt{x_1 x_2} Q, 1/b). \quad (9)$$

If b is small, radiative corrections will be small regardless of the values of x because of the small α_s . When b is large and $x_1 x_2 Q^2$ is small, radiative corrections are still large in T_H , while φ will suppress these regions. But with b increasing, α_s becomes rather large (for example, α_s is large than unity when b is large than $0.5/\Lambda_{QCD}$ GeV⁻¹ for $x_1 = 0.01$, $x_2 = 0.01$ and $Q = 2$ GeV, see Fig. 1) and accordingly the perturbative calculation loses its self-consistency. Therefore, a cut-off on α_s is made to evaluate perturbative contributions and to justify the self-consistency of perturbative calculation. That is to say, if 50% of

the result come from the regions where α_s is not very large (say, < 0.7), the perturbative calculation can be trusted.

Strictly speaking, the perturbative predictions to the regions where α_s is larger than unity are unreliable, although these regions are suppressed. In fact, in the regions of small $x_1 x_2 Q^2$ and large b , the multi-gluon exchange is important and the transverse momentum intrinsic to the bound state wave-functions flows through all the propagators. To respect this point, instead of eq. (9) we suggest that

$$t = \max(\sqrt{x_1 x_2} Q, 1/b_F), \quad (10)$$

where

$$b_F = \begin{cases} b & \text{if } 1/b \geq \sqrt{\langle \mathbf{k}_T^2 \rangle} \\ 1/\sqrt{\langle \mathbf{k}_T^2 \rangle} & \text{if } 1/b < \sqrt{\langle \mathbf{k}_T^2 \rangle}, \end{cases} \quad (11)$$

where $\sqrt{\langle \mathbf{k}_T^2 \rangle}$ is the average transverse momentum. With such a choice, the running coupling constant will be frozen at $\alpha_s(t = \sqrt{\langle \mathbf{k}_T^2 \rangle})$ when b is large and $x_1 x_2 Q^2$ is small. In this way, the perturbative contributions to the pion form factor can be calculated from the present energy with a reasonable α_s , and it should be emphasized that the average transverse momentum $\sqrt{\langle \mathbf{k}_T^2 \rangle}$ always associates with the hadronic wave function

The pion wave function: According to Brodsky-Huang-Lepage prescription [7], one can connect the equal-time wave function in the rest frame and the light-cone wave function by equating the off-shell propagator in the two frames. They got the wave function [7,3] at the infinite momentum frame from the harmonic oscillator model at the rest frame [8]

$$\psi^{(a)}(x, \mathbf{k}_T) = A \exp \left[-\frac{\mathbf{k}_T^2 + m^2}{8\beta^2 x(1-x)} \right], \quad (12)$$

where $m=0.289$ GeV, $\beta = 0.385$ GeV, $A = 32$ GeV⁻¹ are parameters which are adjusted [9] by using the constraints derived [7] from $\pi \rightarrow \mu\nu$ and $\pi^0 \rightarrow \gamma\gamma$ decay amplitudes:

$$\int_0^1 dx \int \frac{d^2 \mathbf{k}_T}{16\pi^3} \psi(x, \mathbf{k}_T) = \frac{f_\pi}{2\sqrt{6}}, \quad (13)$$

$$\int_0^1 dx \psi(x, \mathbf{k}_T = 0) = \frac{\sqrt{6}}{f_\pi}, \quad (14)$$

where $f_\pi = 0.133$ GeV is the pion decay constant.

The mean squared transverse momentum is defined as

$$\langle \mathbf{k}_T^2 \rangle = \int \frac{d^2 \mathbf{k}_T}{16\pi^3} dx |\mathbf{k}_T|^2 |\psi(x, \mathbf{k}_T)|^2 / P_{q\bar{q}}, \quad (15)$$

where

$$P_{q\bar{q}} = \int \frac{d^2 \mathbf{k}_T}{16\pi^3} dx |\psi(x, \mathbf{k}_T)|^2 \quad (16)$$

is the probability of finding $q\bar{q}$ Fock state in the pion. For $\psi^{(a)}(x, \mathbf{k}_T)$, $\langle \mathbf{k}_T^2 \rangle = (0.356 \text{ GeV})^2$. Expressing $\psi^{(a)}(x, \mathbf{k}_T)$ in the b -space, we obtain

$$\phi^{(a)}(x, 1/b) = \frac{2A\beta^2}{(2\pi)^2} x(1-x) \exp\left(-\frac{m^2}{8\beta^2 x(1-x)}\right) \exp\left(-2\beta^2 x(1-x)b^2\right). \quad (12')$$

In order to fit the experimental data and to suppress the end-point contributions for the applicability of pQCD, a model for the pion wave function has been proposed in Refs. [10,11] by simply adding a factorizing function $(1-2x)^2$ to $\psi^{(a)}(x, \mathbf{k}_T)$. It leads to a wave function [14]

$$\psi^{(b)}(x, \mathbf{k}_T) = A (1-2x)^2 \exp\left[-\frac{\mathbf{k}_T^2 + m^2}{8\beta^2 x(1-x)}\right], \quad (17)$$

and

$$\phi^{(b)}(x, 1/b) = \frac{2A\beta^2}{(2\pi)^2} x(1-x)(1-2x)^2 \exp\left(-\frac{m^2}{8\beta^2 x(1-x)}\right) \exp\left(-2\beta^2 x(1-x)b^2\right). \quad (17')$$

In the same way as in eq. (12), the parameters are adjusted to be $m=0.342$ GeV, $\beta=0.455$ MeV, $A=136 \text{ GeV}^{-1}$, and $\langle \mathbf{k}_T^2 \rangle = (0.343 \text{ GeV})^2$.

Numerical calculations: Numerical evaluations for the pion form factor with $\phi^{(a)}$ and $\phi^{(b)}$ are plotted in Fig. 2. We can find the perturbative predictions are still smaller than the experimental data. It is expected to take into account the contributions from higher orders and higher Fock states to reach the data at the intermediate energy.

To evaluate the effects due to the errors in the $s(\xi, b, Q)$ expression, we adopt the formalism of Ref. [4] in our numerical calculations. That is, we choose t as defined in eq. (9) and

neglect the evolution of ϕ with $1/b$. In addition, the same two models of the distribution amplitudes in Ref. [4] are used: (a) the asymptotic wave function [12]

$$\phi^{as}(x) = \frac{3f_\pi}{\sqrt{2N_c}}x(1-x). \quad (18)$$

(b) the Chernyak-Zhitnitsky wave function [13]

$$\phi^{CZ}(x) = \frac{15f_\pi}{\sqrt{2N_c}}x(1-x)(1-2x)^2, \quad (19)$$

where $N_c = 3$ is the number of colors and $f_\pi = 0.133$ GeV the pion decay constant. We find that the corrected expression (eq. (5)) increases the evaluation of the pion form factor by a factor of about 0.8% for the ϕ^{as} and about 1.0% for the ϕ^{CZ} at $Q = 20\Lambda_{\text{QCD}}$. And the effect increases with Q decreasing (reaching about 2.0% for ϕ^{as} and 3.0% for ϕ^{CZ} at $Q = 10\Lambda_{\text{QCD}}$). The effects are sizable individually for the fourth and sixth term in the $s(\xi, b, Q)$ expression, but fortunately they cancel each other in the final expression. As a result, the whole effects on the pion form factor are small.

Summary: In this paper, we re-examine the perturbative pion form factor with the Sudakov suppression. It is found that in the previous formalism there are regions where α_s is larger than unity and the perturbative predictions are still unreliable although these regions are suppressed. Thus a cut-off on α_s has to be made to guarantee the applicability of the pQCD. Observing that in the above regions the multi-gluon exchange is important, we suggest that the running coupling constant should be frozen at $\alpha_s(t = \sqrt{\langle \mathbf{k}_T^2 \rangle})$ when b is large and $x_1x_2Q^2$ is small by taking into account the average transverse momentum $\sqrt{\langle \mathbf{k}_T^2 \rangle}$. In this way, the perturbative contributions to the pion form factor can be calculated from the present energy with a reasonable α_s . The essential point of our scheme is that the running coupling constant “frozen” is determined by the average transverse momentum $\sqrt{\langle \mathbf{k}_T^2 \rangle}$ which always associates with the hadronic wave function. In addition, we correct the previous calculations about the Sudakov suppression factor which plays an important role in the perturbative predictions for the pion form factor.

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FIGURE CAPTIONS

Fig. 1. The evolution of α_s with b for $x_1 = 0.01, x_2 = 0.01$, $Q = 2$ GeV and $\Lambda_{QCD} = 100$ MeV. The solid line is evaluated with eq. (9). The dashed line is evaluated with eq. (10) for $\phi^{(a)}$.

Fig. 2. The pion form factor with $\phi^{(a)}$ (solid line) and $\phi^{(b)}$ (dashed line).